SIMULATION PROCESS MODEL OF THE MOMENTS IN WHICH DAMAGES TO THE MEANS OF TRANSPORT OCCUR

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Summary

It has been attempted in this paper to elaborate a model of the moments in which damages to the means of transport occur in the process of their use. A damage to a technical object has been defined as exceeding admissible limiting values by significant values of the features describing its elements.

On the basis of the analysis of the moments in which damages to the bus subsystems occur and of the time interval lengths between them a simulation model representing a real stream of the damages was built, which enables to evaluate influence of the efficiency of the performed repairs of the means of transport.

Keywords: transport system, mean of transport, model, damage

1. Introduction

It has been attempted in this paper to elaborate a model of the moments in which damages to the means of transport occur in the process of their use.

A damage to a technical object has been defined as exceeding admissible limiting values by significant values of the features describing its elements.

On the basis of relevant references analysis and the results received from our own investigations was found that the damages to the means of transport used in the transport systems result from various forcing factors affecting them.

Some number of the damages results from natural wear of the means of transport elements, which is a natural phenomenon, while the remaining damages may be caused by an inefficient repair of the previous damage. This leads to so called secondary damages to the repaired element, occurred within a short time interval, which is a proof of inappropriate organization of the repairs, poor training level of the repairing teams, limits related to pre and after repair diagnosis, etc.

Within the framework of the operation and maintenance investigations performed in a real operation and maintenance system of the means of transport the time intervals occurring between the successive damages to the means of transport and the moments in which they occur were analysed.

When applying statistical analysis of the moments in which the damages to the means of transport occur, a difference between theoretical and empiric distribution of the time interval values occurring between these moments (Fig.1) was found. A significant difference between the theoretical and empiric distribution appears at the beginning of the interval \((0, t_p)\), and then from the moment \(p\) it reduces to zero. However, the theoretical function is consistent with the empiric distribution in the interval \((t_p, \infty)\). This discrepancy is caused by so called secondary damages resulting from inappropriate repair quality of the damaged elements which occur within this inter-
The investigations performed prove that the secondary damage moments are included within the interval from 0 to 7 days (figure 1).

The analysis of the empirical data (length of the time intervals between the damages) indicates that it is reasonable to describe the probability distribution of the correct work times with a reliability function $R(x)$ formulated as follows [2, 3]:

$$R(x) = pe^{-\lambda x} + (1 - p)R_w(t) \quad (1)$$

It is a mixture of an exponential distribution $pe^{-\lambda x}$ (of unknown parameters value $(p, \lambda)$ with a reliability function described with the relation (1) is a complex problem.

Assuming that for unknown distribution (of the correct work times) gathered within a limited time interval $(0, t_p)$ it is possible to assess the values of the parameters $p$ and $\lambda$, then for high values $t$ it is assumed that: $R(t) = p \exp(-\lambda t)$. Then by applying a method of linear regression (in semilogarithmic graph) it is possible to evaluate the values of the parameters $p$ and $\lambda$ for various random samples cut off from the bottom. A standard regression error – $S(i)$ is calculated for each such approximation, where $i$ stands for an index of a day from which the data are analysed. The analysis of $S(i)$ depending on $i$ value indicates that there is a minimum $s(i)$ for various $i$, mostly for $i = 5, 6, 7, ..., 12$.

![Fig. 1. Changes of the exponential and real functions in the time $t$](Source: own investigations)

The real function flow may be described with a mixture of a probability distribution of the density $g(t)$ with an exponential distribution.

Let $\tau(k)$, where $i = 0, 1, 2, ..., \tau(k) = 0, k = 0, 1, 2, ..., n$ represents the stream (moments) of the damages to the $k$-th technical object.

The difference $\tau_{i+1}(k) - \tau_i(k)$ for $i = 0, 1, 2, ...,$ is the time interval length between $i+1$-st and $i$-th damage to the $k$-th technical object.

$Y_i(n)$ denotes the superposition $n$ - of the damage streams.

Let $X_i(n) = Y_i(n) - Y_{i-1}(n)$, where $i = 0, 1, 2, ..., Y_0 = 0$.
It is assumed that the distribution of the random variable $X_i(n)$ does not depend on $i$.

From the Grigelionis’ theorem it is known that for $n \to \infty$ the random variable $X(n)$ has exponential distribution.

It is assumed that the density of the random variable probability $T$ is described as follows:

$$f(t) = \alpha \cdot g(t) + (1 - \alpha)e^{-\lambda t} \text{ for } f(t) \geq 0$$ (2)

It is a mixture of the probability distribution of the density $g(t)$ with the exponential distribution of the density formulated with the following relation (3):

$$g(t) = \lambda \cdot e^{-\lambda t}$$ (3)

The estimation of the parameter $\alpha$ and $\lambda$ of the density (2) is based on the assumption that the density $g(t)$ takes the values greater than zero which are relatively low within the range from $(t_p, \infty)$.

The analysis of the operation and maintenance investigation results regarding the moments in which the damages occur proves that a set of the damages may be divided into the subsets of primary and secondary damages.

It results from the fact that the successive moments of the damages to the same subsystems are gathered sequentially after a single damage has occurred.

The figure 2 shows an exemplary stream of the damages to a selected subsystem of a mean of transport.

*Fig. 2. Time intervals between the primary and secondary damages*

**Source:** own investigations

$t_i$ – moments in which the primary damages occur,

$t_{ij}$ – moments in which the secondary damages occur,

$T_i$ – time intervals between the moments in which the primary damages occur,

$T_{ij}$ – time intervals between the moments in which the secondary damages occur. As it can be seen in the figure 2, the first of the damages which occurred in the moments $t_i$, cause the sequences of the successive damages to the same subsystem within short time intervals. These damages are called primary ones. While, the damages that follow them, with finite number of repetitions, and occur in the moments $t_{ij}$, are called secondary damages. On the basis of the analysis of the investigation results it has been found that, in general, the reason for the secondary damages is inappropriate quality of the repairs of the primary damages, subsystem elements. The primary damages do not depend on one another and they occur randomly (they are not related to one another with the cause and effect links). The secondary damages are interdependent, because their occurrence depends on a previous occurrence of a primary damage and on the result of its inappropriate repair or inappropriate repair of the successive secondary damage [6, 9, 10].

It means that the secondary damage occurrence probability $B_{ij}$ conditioned by a primary damage occurrence $A_{ti}$ is greater than the primary damage occurrence probability $A_{ti}$.
Reduction of the conditional secondary damage occurrence probability may be a starting point for reduction of the damage intensity, which leads to the increased level of the performed repairs efficiency. This may be achieved by elimination of those damages which occur due to unreasonable realization of the repair process.

The following definition of the repair efficiency has been adopted on the basis of the analysed references:

"Repair efficiency – is a goal realization degree, in which such a repair should be finished, without considering its economic aspects. The specific repair of the investigated subsystem is considered to be efficient, if it gets closer, in more positive way, to the specified goal being the assurance of the system task ability to accomplish the task within specified time interval" [5, 11].

2. Aim

The aim of this paper is to build a simulation process model of the moments in which damages occur to the means of transport enabling generation of the damage streams, the analysis and evaluation of which will make it possible to undertake reasonable actions aimed at increasing the efficiency level of the repairs being realized.

3. Investigation object

The investigation object are the subsystems of the buses being maintained and operated within an urban transport system. While the investigation subject are the damages to the selected subsystems of the means of transport and the moment in which they occur.

4. Investigation methodology

The investigations were performed within an urban bus transport system. The operation and maintenance investigations concern the damages to the subsystems of the means of transport and the moment in which they occur. These investigations were performed using a passive experiment method in real operation and maintenance conditions. A set consisting of 28 means of transport used in real operation and maintenance conditions was randomly selected for the investigation purposes. The investigation results cover five-year long vehicle operation and maintenance period.

5. Process model of the moments in which the damages occur

Because of impossibility to carry out long-term operation and maintenance investigations as well as due to difficulties to implement changes to the repair process of the means of transport in an urban transport system a simulation model of the moments in which the damages occur was built to facilitate generation of the streams of the damages to the bus subsystems. The analysis and evaluation of the results obtained at the simulation stage will enable to evaluate the efficiency of the repairs being realized in a real transport system.

By analysing the moments in which the damages to the bus subsystems occur and the values of the time intervals between them, the real process parameter values were estimated.

This was the basis for creating a simulation model representing a real stream of the damages, which, when being examined, enables to evaluate the efficiency of the repairs realized within a service process.

The figure 3 shows a dialog box of the simulation programme used to enter the parameter values of a real process to the simulation model.
On the basis of the relevant references and the results of our own investigations the generalized Gamma distribution [3] has been adopted to generate, in the simulation program, the streams of the damages to the individual bus subsystems. By means of the adopted parameter values \((b - \text{scale parameter and}, \ p - \text{shape parameter}, \ v - \text{shape parameter})\) of this distribution, the time intervals between the moments in which the primary damages occur were simulated in the program. In case of adopting the values of the parameter \(v=1\), the time intervals were generated according to the Gamma distribution. However, for the parameters with the values \(p=1\) and \(v=1\) the time intervals were generated according to the exponential distribution.

The random variable \(X\) has gamma distribution, if its density is expressed by the following formula (4) [3]:

\[
f(x) = \frac{1}{b \Gamma(p)} x^{p-1} e^{-\frac{x}{b}} \quad x > 0, \ p, b > 0,
\]

where: \(\Gamma(p)\) is a gamma function expressed with the formula (5):

\[
\Gamma(p) = \int_0^\infty x^{p-1} e^{-x} dx
\]

If \(p = 1\), then we have an exponential distribution as a particular case. In case when the parameter \(p\) is an integer number, then the gamma distribution is the Erlang’s distribution. The Parameter \(p\) is the form (shape) parameter, while the parameter \(b\) is the scale parameter [2, 3].

The average value \(EX\) is expressed with the following relation (6):

\[
EX = pb
\]
and the variation with the formula (7):

\[ D^2 X = p b^2. \] (7)

The equations (6, 7) may form the basis for creating estimators \( \hat{p} \) and \( \hat{b} \) of the parameters \( p \) and \( b \). It results from these equations that:

\[ \hat{b} = \frac{s^2}{x}, \quad \hat{p} = \frac{x}{\hat{b}} = \frac{x}{s^2}. \] (8)

If we define a new variable:

\[ T = x^v, \quad v > 0 \] (9)

then the random variable \( T \) has generalized gamma distribution with the probability density expressed with the relation (10) as follows:

\[ f(t) = \frac{v}{b^p \Gamma(p)} \cdot t^{pv-1} e^{-\frac{t^v}{b}}, \quad t > 0. \] (10)

The density of the generalized gamma distribution is stated in this paper [3] in the following form:

\[ f(t) = \frac{v}{-b \Gamma(p)} \left( \frac{t}{b} \right)^{pv-1} \exp \left\{ -\left( \frac{t}{b} \right)^v \right\}, \quad t > 0. \] (11)

The average value of this distribution is expressed with the formula (12):

\[ ET = b \frac{\Gamma \left( p + \frac{1}{v} \right)}{\Gamma(p)}. \] (12)

and the variation with the formula (13):

\[ D^2 T = b^2 \left\{ \frac{\Gamma \left( p + \frac{2}{v} \right)}{\Gamma(p)} - \frac{\Gamma^2 \left( p + \frac{1}{v} \right)}{\Gamma^2(p)} \right\}. \] (13)

The estimation of this distribution parameter has been stated herein [3].

The random numbers coming from the generalized Gamma distribution (three parameters: \( b, p, v \)) are obtained as random numbers coming from the gamma distribution with the parameters \( b \) and \( p \), which were raised to the \( \frac{1}{v} \) power.
The random numbers out of the Gamma distribution \((b, p)\) were obtained by means of the GAMMAINV function built in the EXCEL 2000 program (it is a realization of the generator of the random numbers out of the Gamma distribution \((b, p)\) by inverting the distribution function of this distribution).

\[
    r(b, p, v) = \left(\text{Gamma}(b, p)\right)^{\frac{1}{v}}
\]

The number of the secondary damages occurring in the sequence of the events after the primary damage (figure 2) was generated according to the Poisson’s distribution with specified value of the parameter \(\lambda\) (expected value) \([1, 2, 3]\).

The random numbers coming from the Poisson’s distribution are obtained by means of the following algorithm:

- \(q = e^{-\lambda}\) is calculated for the determined \(\lambda\)
- \(x=0, S=q\) and \(p=q\)
- a random number \(r\) is generated out of the uniform distribution over the range \([0, 1)\)
- As long as \(r > S\) the following parameters are calculated:
  - \(x=x+1\)
  - \(p=p*\lambda/x\)
  - \(S=S+p\)

When \(S\) becomes greater or equal to \(r\) the value of \(x\) is adopted as a number out of the Poisson’s distribution with the parameter \(\lambda\).

The time intervals between the secondary damages were generated out of the Erlang’s distribution with the defined values of the parameters of this distribution, such as: number of the secondary damages occurred in the sequence after a primary damage and the value of the parameter \(\lambda\).

The random numbers out of the Erlang’s distribution with the parameters \(n\) and \(\lambda\) are obtained on the basis of the following formula (15):

\[
    r(n, \lambda) = \sum_{k=1}^{n} \left( -\frac{1}{\lambda} \ln(rnd) \right)
\]

where:

- \(rnd\) – is a random number out of uniform (rectangular) distribution over the range \([0, 1)\)

The number of the events, in the damage stream generated by the simulation program, was determined on the basis of the statistical analysis of the real streams of the damages to the subsystems of the means of transport.

6. Chosen investigation results and verification of the model adequacy

On the basis of the analysis of a real stream of the damages to the selected bus subsystem, the damages were classified according to the adopted criteria (value of the limiting distance travelled expressed in kilometres and the critical time of correct work) to the subset of the primary or secondary damages. Due to the analysis carried out, the average times of correct work between the damages were determined. The determined statistics were entered to the simulation model (table 1) and the damage streams with similar parameters of the statistics, if compared to the statistics determined on the basis of the empiric data, were generated with its help.
Table 1. Exemplary values of the parameters adopted to simulate a stream of the damages to the electric installation subsystem in a selected bus

<table>
<thead>
<tr>
<th>Name or parameter symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>b – scale parameter</td>
<td>1.08</td>
</tr>
<tr>
<td>p – shape parameter</td>
<td>20</td>
</tr>
<tr>
<td>v - shape parameter</td>
<td>0.897</td>
</tr>
<tr>
<td>λ - number of secondary damages</td>
<td>2.15</td>
</tr>
<tr>
<td>row</td>
<td>1.9</td>
</tr>
<tr>
<td>λ</td>
<td>0.393</td>
</tr>
<tr>
<td>Number of damages</td>
<td>115</td>
</tr>
<tr>
<td>Average time of correct work between the primary damages</td>
<td>22.89641</td>
</tr>
<tr>
<td>Average time of correct work between the secondary damages</td>
<td>4.834606</td>
</tr>
</tbody>
</table>

Source: own investigations

In order to verify consistency of the real damage stream and the one generated by the simulation program, Kolmogorov-Smirnov goodness of fit test being used in case of small number of tests was applied. The Kolmogorov-Smirnov goodness of fit test was used in order to check the consistency of the hypothesis saying that two tests have the same distribution. The differences between both distribution functions were considered in this test. The limiting distribution of the appropriate statistics, used to build the critical range for this test, is the same as limiting distribution of the statistics λ by Kolmogorov [1, 2, 3, 12].

The critical value \( \lambda_{\alpha} \) was read from the table of this distribution for the predefined significance level \( \alpha = 0.95 \) to have \( P\{\lambda \geq \lambda_{\alpha} = \alpha\} \) when the calculated value of the statistics \( \lambda \) meets the inequality \( \lambda \geq \lambda_{\alpha} \). The hypothesis \( H_0 \) was rejected because both tests have different distributions. However in case, when \( \lambda < \lambda_{\alpha} \), there are no grounds to reject the hypothesis saying that the distributions of both tests are the same. The limiting value \( \lambda_{\alpha} \) for the adopted value of the significance level \( \alpha \) is \( \lambda_{\alpha} = 1.358 \) [1, 2, 3, 12].

In case when the distribution consistency was found a damage stream in which the primary and secondary damages existed was simulated according to the previously adopted values of the parameters, as stated in the table 1. Afterwards, the repair efficiency coefficient was determined according to the relation (16).

The value of the repair efficiency index of the \( j \)-th bus subsystem is described with the relation (16) [11]:

\[
WS_j = \frac{N_j(t) - N_j^S(t)}{N_j(t)} = \frac{N_j^S(t)}{N_j(t)} \quad , j = 1,2,...,m
\]

The next step was to simulate, by means of the program, improvement of the efficiency of the repairs being realized by reducing the value of the parameter \( \lambda \) by 25% each time, as it is presented in the table 3.
Table 2. Chosen investigation results of the goodness of fit test for the electric installation of Ikarus IK280 buses

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Bus side number</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2400</td>
</tr>
<tr>
<td>b - scale parameter</td>
<td>1</td>
</tr>
<tr>
<td>p - shape parameter</td>
<td>44</td>
</tr>
<tr>
<td>v - shape parameter</td>
<td>0.9543</td>
</tr>
<tr>
<td>λ - number of secondary damages</td>
<td>2.6</td>
</tr>
<tr>
<td>row</td>
<td>3</td>
</tr>
<tr>
<td>λ</td>
<td>0.487</td>
</tr>
<tr>
<td>Number of events</td>
<td>1000</td>
</tr>
<tr>
<td>Average time of correct work between the primary damages</td>
<td>44.93333</td>
</tr>
<tr>
<td>Average time of correct work between the secondary damages</td>
<td>6.164384</td>
</tr>
<tr>
<td>λ - statistics</td>
<td>1.299007</td>
</tr>
<tr>
<td>Test result</td>
<td>Consistent</td>
</tr>
</tbody>
</table>

Source: own investigations

Table 3. Values of the model parameters adopted to simulate the streams of the damages to the electric installation of a selected bus for different number of secondary damages $L_{uw}$

<table>
<thead>
<tr>
<th>Name or parameter symbol</th>
<th>Value 100% $L_{uw}$</th>
<th>Value 75% $L_{uw}$</th>
<th>Value 50% $L_{uw}$</th>
<th>Value 25% $L_{uw}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>b - scale parameter</td>
<td>1.08</td>
<td>1.08</td>
<td>1.08</td>
<td>1.08</td>
</tr>
<tr>
<td>p - shape parameter</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>v - shape parameter</td>
<td>0.897</td>
<td>0.897</td>
<td>0.897</td>
<td>0.897</td>
</tr>
<tr>
<td>λ - number of secondary damages</td>
<td>2.1</td>
<td>1.15</td>
<td>0.455</td>
<td>0</td>
</tr>
<tr>
<td>n – Row</td>
<td>1.9</td>
<td>1.45</td>
<td>1.04</td>
<td>0.45</td>
</tr>
<tr>
<td>λ - parameter</td>
<td>0.393</td>
<td>0.393</td>
<td>0.393</td>
<td>0.393</td>
</tr>
<tr>
<td>N - Number of damages</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
</tr>
<tr>
<td>Average time of correct work between the primary damages</td>
<td>22.89641</td>
<td>22.89641</td>
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<td>22.89641</td>
</tr>
<tr>
<td>Average time of correct work between the secondary damages</td>
<td>4.834606</td>
<td>4.834606</td>
<td>4.834606</td>
<td>4.834606</td>
</tr>
</tbody>
</table>

Source: own investigations
5. Summary

1. On the basis of the analysis of the results contained in the table 3 it may be stated that the simulation model enables to generate a damage stream with the values of the statistics being similar to a real damage stream, which is a prove of its adequacy.

2. The model of the moments in which the damages occur presented herein meets the identified goals and may also be to generate the streams of the damages to the subsystems of the means of transport in other real transport systems as well. The analysis of the simulation investigation results makes it possible to evaluate the efficiency degree of the realized repairs with no need to carry out long-term operation and maintenance investigations.

3. Using the model proposed in this paper facilitates to simulate various variants of improving the efficiency of the realized repairs of the means of transport (table 3), which will enable the transport system decision makers to undertake appropriate actions intended to control the repair process in a real transport system in a reasonable way.

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